The Hubble Law & The Structure of the Universe

The Hubble Law is a relationship between two quantities – the speed of and distance to a galaxy. In order to determine the Hubble Law and to measure Hubble’s constant, we need independent measurements of both of these quantities. Speed is somewhat easier to measure (think Doppler), but distance is pretty tough. In the two sections below, we’ll look at two datasets used to determine the Hubble Law – Hubble’s own data from 1929, and the data made possible with the Hubble Space Telescope in the late 1990’s.

Hubble’s Hubble Law

Edwin Hubble determined the law that bears his name from data presented in a journal paper in 1929. Those data are shown in graphical and tabular form on the first sheet of the “HubblesData.xlsx” Excel workbook.

Hubble plotted the distance and velocity data with the distance in Mpc (“mega-parsecs”) on the x-axis and velocity in units of km/s on the y-axis. From the plot, he concluded that he could draw a straight line through these data and derive a linear relationship between the recession velocity and the distance. The relationship described by that line is

\[ \text{speed} = H_0 \times \text{distance} \]

Here, \( H_0 \) (usually pronounced “aitch-naught”) is the slope of the line and is also referred to as “Hubble's constant.” The subscript “0” means that it is the slope we measure today. In principle, the slope can change with time.

Examine Hubble’s data, and estimate the value for \( H_0 \):

Measurement: \( H_0 = \) _________ (estimated)

What are the units for \( H_0 \)? __________

You can ask Excel to calculate a best-fit straight line to these data by right-clicking on any data point in the plot and then selecting “Add Trendline.” In the dialog box that comes up, choose a linear trend, and check “Set intercept = 0.0” and “Display Equation on chart.”
Question: Why should we require that the y-intercept of our fit be zero?

Measurement: $H_0 = \text{__________} \text{ (from Excel trendline)}$

Was your estimate close to the fitted value?

Hubble reasoned that this relation, if verified, could be used to predict the distances to any faraway galaxy from just a Doppler measurement of its recession velocity. Even back then, Doppler measurements were much easier and more accurate than any distance measurement over large distances.

Calculation: Using the best-fit value for $H_0$, determine the distance to a galaxy whose recession velocity is $1000 \text{ km/s}$.

Look at the individual data points in the plot. There are several galaxies with recession velocities of about $1000 \text{ km/s}$. Based on the scatter of the data in Hubble’s plot, estimate the uncertainty in the distance you calculated. (Hint: Consider a phantom galaxy that has a measured velocity of say, $1000 \text{ km/s}$. Looking at the plotted data, at what distance could this galaxy possibly be, and still “fit in” with the rest of the dataset? Use that information to gauge the potential uncertainty of your result above.)
Today’s Hubble Law

Frankly, it was pretty gutsy for Hubble to draw a straight line through his pretty lousy data. After all, given the scatter in his data, he could have probably fit a curved line, like a parabola or an exponential, just as well. As we’ll see later in the lab, he had other reasons for preferring a straight line. And, as it turns out, he was right.

The galaxy distances and velocities based on the Hubble Space Telescope (HST) key project on the Extragalactic Distance Scale are displayed on the second sheet of the Excel workbook. Take a close look at the plotted data.

**Question:** How does this dataset differ from Hubble’s original data? *(Hint: Look at the axes.)*

You can fit a linear trendline to these data in the same way you did for the first dataset.

**Measurement:** $H_0 = \underline{\hspace{2cm}}$ (from Excel trendline fit to HST data)

How does this value of $H_0$ compare to Hubble’s original value?

You should be concerned right now, because the two datasets give pretty different results (if you didn’t find that, talk to your instructor or TA). Something must be very different between Hubble’s original dataset and the modern HST dataset. We can try to figure out what’s going on here by examining the two datasets carefully. There are several galaxies common to both datasets. Find five of them, and examine the distance and velocity data for both the old (Hubble) and new (HST) datasets:

<table>
<thead>
<tr>
<th>Galaxy Name</th>
<th>Distance (from old data)</th>
<th>Velocity (from old data)</th>
<th>Distance (from new data)</th>
<th>Velocity (from new data)</th>
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The linear relationship Hubble found has been confirmed many times in the intervening 80 years, though our determination of the constant $H_0$ has changed considerably as we have improved our ability to measure distances. The currently accepted value is some 7 times lower than the value Hubble derived from his early data. Note that this does not mean that universe is expanding more slowly now than it was in 1929 — the difference in the value of $H_0$ is due entirely to improvements in measurement.

Nonetheless, the data (even Hubble’s original data) clearly show that the Universe is expanding, and that tomorrow the distances between galaxies will be greater than they are today. In the next section, we’ll see how a Hubble Law naturally results from an expanding Universe.

**A Universe on a Tabletop**

On the tabletop before you is a one-dimensional model of a universe. It’s also a slinky, but today it will serve well as an expanding universe. Pull the slinky until it is stretched out about 60 cm or so, and make sure that it is evenly stretched out (i.e., no bunching anywhere). This is the extent of your universe at a time $t_0$. Attach paper clips to single coils of the slinky wherever you’d like. Put a total of six paper clips on the slinky. These are the galaxies in your universe.

Choose one of the galaxies to be your home, and using the meter stick, measure the distance from your home galaxy to each of the other galaxies in the universe. Note that since you’re measuring the distance from your location outward to other places in the universe, those distances are positive regardless of the direction. Record the distances in Table 1.

**Question:** Which kind of measurement -- distance or velocity -- is most discrepant between the two datasets? __________

Why is it harder to measure one of these quantities than the other?
Table 1 Your Own Model Universe

<table>
<thead>
<tr>
<th>Source Name</th>
<th>Distance at time = $t_1$ (cm)</th>
<th>Distance at time = $t_2$ (cm)</th>
<th>Speed (cm/yr)</th>
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Now stretch your slinky so that it spans a distance of 90–120 cm in total. This is now the size of your Universe at a time $t_2$, which we’ll say is one year later than $t_1$. Measure the positions from your home galaxy to each of the other galaxies in the universe and record the results in the appropriate column in the Table 1. You can now calculate the speed of each galaxy, since you know how far it has traveled in the time interval $(t_2 - t_1)$. Calculate this velocity for each galaxy and put your results in the table above.

Now plot the data for this model universe in Figure 1 just as Hubble plotted his data in the 1929. Put the distance at time $t_2$ on the x-axis and the speed on the y-axis.

**Figure 1. Hubble diagram for your model universe.**
Note that you’ve just come up with the same type of Hubble Law relationship for paper clips on a slinky that Hubble found for galaxies in the Universe. The important connection between the model and the real Universe is that every part of the slinky (or Universe) participates equally in the expansion; that is, the slinky stretches evenly across its length. If this were not the case (e.g., maybe you tied an extra rubber band between two of the paper clips so that they would separate more slowly than the rest of the slinky), you would not get a linear relationship between velocity and distance.

### Is Our Location Really Special?

One of the biggest philosophical arguments against the idea of an expanding Universe is the implication that we are somehow at the center, in a special location. Let’s examine how accurate that perception is. Transplant yourself to another galaxy (that is, choose another paper clip to be your home). Using the data from the above table, calculate the distances to every other galaxy in the Universe from your new home at times $t_1$ and $t_2$. (Note: You shouldn’t have to make any new measurements.) Complete Table 2 and Figure 2 to construct a Hubble diagram for these new data.

**Measurement:** Draw a straight line through your data and calculate the constant $H_0$ for your tabletop universe (including appropriate units).

**Question:** If there were a paper clip at a distance of 3 meters from your home galaxy, how fast would it be moving?
Table 2. Your model universe from a different point-of-view.

<table>
<thead>
<tr>
<th>Source Name</th>
<th>Distance at time = ( t_1 ) (cm)</th>
<th>Distance at time = ( t_2 ) (cm)</th>
<th>Speed (cm/yr)</th>
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Checkpoint: Distances are measured without regard to direction, and so there should be no negative (−) values of the distance between galaxies.

Figure 2. A revised Hubble diagram for your model universe.
It turns out that no matter where you are in an expanding universe, you see the same Hubble Law relationship. This removes a major philosophical hurdle from the expanding Universe idea because even though it may look like you are the center of the expansion, it *looks like that from any vantage point in the Universe.* That means that our location isn’t special at all.

**Using the Hubble Law as a Tool for Measuring Large Distances**

While the idea that the universe is expanding in all directions is pretty mind-blowing, it’s also really useful. If there’s a direct linear relationship between the recession speed of a galaxy and its distance from us, then we can use that relationship to measure the distances to really faraway galaxies. And measuring the recession speed just isn’t that hard, even for really distant galaxies. We just need to measure a galaxy’s spectrum, identify spectral lines in the spectrum, and use the Doppler relation to infer the speed from the shift in the wavelengths of those spectral lines.

For the next part of the lab, we will use WorldWide Telescope (WWT), a multi-perspective visualization tool. We will use it to identify galaxies and retrieve their spectra from online databases. The software involves both video and audio output, so be sure that your computer’s audio output is not muted, and have your ear buds plugged in. As you work through the interactive “tour,” follow along with this writeup and record your notes and responses when prompted.

Your instructor will explain how to launch the first part of the WWT tour. There is an audio track, so please make sure you have plugged in your earbuds. *Don’t exit the tour when it’s done.*

When the tour ends and you’re prompted to measure the redshift for several galaxies, open up the Excel file “GalaxyRedshiftCalculator.xls.” The first tab in the spreadsheet lists 50 galaxies from the Sloan Digital Sky Survey (SDSS). We will determine the redshift for one of these galaxies as follows:

1. Pick a number between one and 50, and choose the galaxy with that “BU” number.
2. Ask you instructor or TA how to enter the Right Ascension and Declination coordinates into WWT to slew to this galaxy. Once you’ve entered them correctly, WWT will slew to reveal a fuzzy, galaxy-like blob in your view.
3. Right click to bring up the WWT Finder Scope, and left-click and drag to center the scope on the galaxy.

**Measurement:** Calculate the new value for $H_0$ and enter it here, with units.
4. Click on the “Research” button on the Finder Scope, and select “Information” and then “Look up on SDSS.” This should bring up a browser window showing both an image and the spectrum for your galaxy.

5. Click on the “Get Spectrum as CSV” link under the spectrum plot. This should download a file that can be opened with Excel.

6. Open that file, select all data (Control-A), and then copy (Control-C).

7. Switch to the “GalaxyRedshiftCalculator” Excel file, and paste (Control-V) the data into the second tab in the pink-colored block labeled “paste spectrum data here.”

8. Switch to the third tab in the Excel file (labeled “Redshift Determination”) to see the galaxy’s spectrum.

9. To determine the redshift for this galaxy, we’ll need to identify some of the absorption lines in the spectrum. Follow the directions on the “Redshift Determination” tab to measure the wavelengths of the shifted spectral lines for your chosen galaxy.

Congratulations! You’ve just determined the redshift for one of our universe’s galaxies. Enter the redshift in the table below, and use the same procedure to determine the redshift for two more galaxies in the list on the first tab of the “GalaxyRedshiftCalculator” spreadsheet.

<table>
<thead>
<tr>
<th>Galaxy “BU” name</th>
<th>Redshift (z)</th>
<th>Recession Velocity (km/s)</th>
<th>Distance (Mpc)</th>
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Using the redshifts you determined, you can calculate the recession velocities for each galaxy using the standard Doppler relation:

\[ z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} \]

Then, with the recession velocity of the galaxy, you can determine its distance using the Hubble relation:

\[ v = H_0 d \]

where, for our universe, \( H_0 = 73 \text{ (km/s)/Mpc} \).

Now that you know how we can calculate the distances to galaxies just by observing their spectra, let’s see what we can learn about the large scale structure of our universe. Launch the last portion of our WWT tour by following your instructor’s directions.